Abstract

The problem of polynomial representability of functions is central to many branches of mathematics. If the underlying set is a finite field, every function can be represented as a polynomial. In this thesis we consider polynomial representability over a special class of finite rings, namely, $\mathbb{Z}_{p^n}$, where p is a prime and n is a positive integer. This problem has been studied in literature and the two notable results were given by Carlitz (1965) and Kempner (1921). While the Kempner's method enumerates the set of distinct polynomial functions, Carlitz provides a necessary and sufficient condition for a function to be polynomial using Taylor series. Further, these results are existential in nature.

The aim of this thesis is to provide an algorithmic characterization, given a prime p and a positive integer n, to determine whether a given function over $\mathbb{Z}_{p^n}$ is polynomially representable or not. Note that one can give an exhaustive search algorithm using the previous results. Our characterization involves describing the set of polynomial functions over $\mathbb{Z}_{p^n}$ with a 'suitable' generating set. We make use of this result to give an non-exhaustive algorithm to determine whether a given function over $\mathbb{Z}_{p^n}$ is polynomial representable.